GUJARAT UNIVERSITY

B. E. Semester-1 (All Branch except Aero.)

Question Bank

Each question has 10 marks.

Q.1	Find the rank of (i) $\begin{bmatrix} 6 & 1 & 1 & 1 \\ 16 & 1 & -1 & 5 \\ 7 & 2 & 3 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$
Q.2	Reduce the following matrix to normal form and find its rank. (i) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$
Q.3	Find inverse of a matrix by Gauss Jordan Reduction method (i) $\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$
Q.4	Examine for consistency the following equations and solve them if they are consistent: (i) $\begin{aligned} 5x+3y+7z&=4 \\ 3x+26y+2z&=9 \\ 7x+2y+10z&=5 \end{aligned}$ (ii) $\begin{aligned} 2x-y-z&=7 \\ 3x-2y-2z&=10 \\ 12x-y-7z&=19 \end{aligned}$
Q.5	Find the eigenvalues & eigenvectors of the matrix (i) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 6 \end{bmatrix}$
Q.6	Evaluate (i) $\int_0^{\pi} (1+\cos\theta)^4 \, d\theta$ (ii) $\int_{-2\pi}^{2\pi} \sin^6 x \, dx$
Q.7	(i) Prove that $\int_0^{\pi} x\cos^6 x \, dx = \frac{5\pi^2}{32}$ (ii) $\int_0^{\pi} x^6 \sin^{-1} x \, dx = \frac{\pi}{14} - \frac{16}{245}$
Q.8	Evaluate (i) $\int_0^{\pi} \sin^2\theta (1+\cos\theta)^4 \, d\theta$ (ii)
Q.9	If $f_n = \int_0^{\pi/4} \tan^n x \, dx$ show that $f_n + f_{n-2} = \frac{1}{n-1}$ and the deduce the value of f_5.
Q.10	If $f_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx, (n>2)$ show that $f_n + f_{n-2} = \frac{1}{n-1}$ .Hence evaluate $\int_{\pi/4}^{\pi/2} \cot^6 x \, dx$.
Q.11	Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to one extremity of the latus rectum.
Q.12	Find the perimeter of the cardioid $r = a(1+\cos\theta)$

GUJARAT UNIVERSITY

B. E. Semester-1 (All Branch except Aero.)

Question Bank

Each question has 10 marks.

Q.13	Find the whole length of lemniscate $r^2 = a^2 \cos 2\theta$.
Q.14	Find the area between the curve $y^2(2a - x) = x^3$ and its asymptote.
Q.15	Find the area of the loop of the curve $x^3 + y^3 = 3axy$.
Q.16	Evaluate (i) $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$ (ii) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$
Q.17	Evaluate (i) $\lim_{x \to 0} \frac{x - \tan x}{x^3}$ (ii) $\lim_{x \to 0} \frac{a^x - b^x}{x}$
Q.18	Evaluate (i) $\lim_{x \to 0} \log_{\tan x} \tan 2x$ (ii) $\lim_{x \to 0} \frac{\cot x}{\cot 2x}$
Q.19	Evaluate (i) $\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$ (ii) $\lim_{x \to 0} \left(\frac{1}{x^2} - \cot^2 x \right)$
Q.20	If $y = \frac{x^4}{(x-1)(x-2)}$, find y_n.
Q.21	Find the n^{th} derivative of (i) $\tan^{-1}\left(\frac{1-x}{1+x}\right)$ (ii) $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$
Q.22	If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$
Q.23	If $y = \cot^{-1} x$ prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$
Q.24	If $y = (x^2 - 1)^n$, prove that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$
Q.25	If $y = e^{m\cos^{-1} x}$ show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0.$
Q.26	Expand $f(x) = \sec x$ in power of x up to x^4 by maclaurin`s series.
Q.27	Prove that $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right)$
Q.28	Test the convergence of the Sequences $\frac{\ln n}{n}, n^{\frac{1}{n}}$

Each question has 10 marks.

Q.29	Find the sum of the series if it converges. (i) $\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\ldots+\frac{1}{(n+1)(n+2)}+\ldots$ (ii) $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots+\frac{(-1)^{n-1}}{2^{n-1}}+\ldots\ldots\ldots$
Q.30	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^2+1}, \sum_{n=1}^{\infty}\frac{1}{1+2+3+4+\ldots\ldots+n}$
Q.31	Discuss the convergence of $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n, \sum_{n=1}^{\infty}\left(\frac{n-1}{n}\right)^n$
Q.32	Discuss the convergence of p-series.
Q.33	Discuss the convergence of (a) $\sum_{n=1}^{\infty}\frac{\ln n}{n^{3/2}}$ (b) $\sum_{n=1}^{\infty}\frac{\tan^{-1} n}{n^{1.1}}$
Q.34	Discuss the convergence of (a) $\sum_{n=1}^{\infty}\frac{n^{\sqrt{2}}}{2^n}$ (b) $\sum_{n=1}^{\infty} n!e^{-n}$ (c) $\sum_{n=1}^{\infty}\frac{n}{(\ln n)^n}$
Q.35	Discuss the convergence of (a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ (b) $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{10}\right)^n$
Q.36	Verify that $w_{xy}=w_{yx}$ Where $w=e^x+x\ln y+y\ln x$, w =sinx.cosy + cosx.siny
Q.37	Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as function of u and v both by using the Chain Rule $z=4e^x\ln y,\quad x=\ln(u\cos v),\quad y=u\sin v$

Each question has 10 marks.

Q.38	Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as function of u and v both by using the Chain Rule $z=\tan^{-1}\left(\frac{x}{y}\right), \quad x=u\cos v, \quad y=u\sin v$
Q.39	Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as function of u and v both by using the Chain Rule $w=xy+yz+xz, \quad x=u+v, \quad y=u-v, \quad z=uv; \quad (u,v)=(1/2,1)$
Q.40	Find the tangent plane normal line at the point P_0 on the given surfaces. $z^2-2x^2-2y^2-12=0 \qquad P_0(1,-1,4)$ $x^2+2xy-y^2+z^2=7 \qquad P_0(1,-1,3)$
Q.41	Find the tangent plane normal line at the point P_0 on the given surface. $\cos\pi x-x^2y+e^{xz}+yz=4 \qquad P_0(0,1,2)$
Q.42	Find an equation for the plane that is tangent to the given surface at the given point. $z=\ln(x^2+y^2), \qquad (1,0,0)$ $z=e^{-(x^2+y^2)}, \qquad (0,0,1)$
Q.43	The volume $V=\pi r^2h$ of a right circular cylinder is to be calculated from measured values of r and h. Suppose that r is measured with an error of no more than 2% and h with an error of no more than 0.5%. Estimate the resulting possible percentage error in the calculation of V.
Q.44	Find the linearization $L(x,y)$ of the function at each point. $f(x,y)=3x-4y+5 \;\; at \quad (1,1)$ $f(x,y)=e^x\cos y \;\; at \quad (0,\pi/2)$
Q.45	Find the local extreme values of the function $f(x,y)=xy-x^2-y^2-2x-2y+4$
Q.46	Find local maxima & local minima and saddle points of the function $f(x,y)=x^2+xy+3x+2y+5$
Q.47	Find local maxima & local minima and saddle points of the function

GUJARAT UNIVERSITY

B. E. Semester-1 (All Branch except Aero.)

Question Bank

Each question has 10 marks.

	$f(x,y)=x^2-y^2-2x+4y+6$
Q.48	Find local maxima & local minima and saddle points of the function $f(x,y)=x^2+2xy$
Q.49	(a) Express the following complex numbers into Polar form a) 1+√3i b)1+√2+i (b) If x and y are real ,solve the following equation: $\frac{iy}{ix+1}-\frac{3y+4i}{3x+y}=0$
Q.50	Simplify $\frac{(\cos 3\theta+i\sin 3\theta)^4(\cos 4\theta-i\sin 4\theta)^5}{(\cos 4\theta+i\sin 4\theta)^3(\cos 5\theta+i\sin 5\theta)^{-4}}$ Prove that $(\frac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta})^4=\cos 8\theta+i\sin 8\theta$
Q.51	Prove that $(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta})^n=\cos(\frac{n\pi}{2}-n\theta)+i\sin(\frac{n\pi}{2}-n\theta)$
Q.52	Prove that n^{th} roots of unity form a G.P. and sum of the roots is zero and product of the roots is $(-1)^{n-1}$
Q.53	Solve the equations $x^7+x^4+x^3+1=0$, $x^4-x^3+x^2-x+1=0$ using De Moivre's theorem.
Q.54	(a) Find the solution to the differential equation $x^2\frac{dy}{dx}=(x+1)(y+1)$ given that y = 2e when x = 1 (b) Show that y=acosx+sinx is asolution of cosx$\frac{dy}{dx}+y\sin x=1$
Q.55	(a) Solve : $\frac{dy}{dx}+2xy=3x^2$, (b) Solve : $(3x-2)\frac{dy}{dx}=y^2$
Q.56	Solve $(1+e^{\frac{x}{y}})dx+e^{\frac{x}{y}}(1-\frac{x}{y})dy=0$
Q.57	(a) Solve : $\frac{dy}{dx}+3y=2x-1$, (b) Solve $x\frac{dy}{dx}+2y=\log x$
Q.58	(a) Evaluate $\int_0^\infty x^{3/2}\, e^{-x}\, dx$ (b) Evaluate $\int_0^1 x^4\,(1-x)^3\, dx$

Each question has 10 marks.

Q.59	Find $\Gamma(-\frac{1}{2})$